International Journal of Theoretical & Applied Sciences, **4**(2): 66-71(2012)

ISSN No. (Print) : 0975-1718
ISSN No. (Online) : 2249-3247

VMI Model To Predict K-Values In SD Nuclei

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(Received 01 September, 2012, Accepted 15 October, 2012)

ABSTRACT: Super deformation of nucleus is defined as *"a nucleus that is very far from spherical shape forming an ellipsoidal shape with axes ratio, approximately 2:1:1"*. During nuclear super deformation, energy differences occur between different bands in ground state and excited state. Super deformed nuclei are unstable and can explain the process of nuclear decay and half lives of atoms. Evidences of super deformation are obtained through Gamma energy differences in different bands through spectroscopic studies. Band head energy (E_0) and angular momentum (K) value of SD nucleus is not known yet. This study evaluate the features of super deformed nucleus in different mass regions A = 130, 150, and 190 in even-even and odd-odd SD nuclei. VMI model equation was used to estimate angular momentum (K value) in terms of softness parameter (σ) and Mallman's energy ratios (R) for SD bands. The σ is calculated by fitting experimentally known Gamma energies with spin by using best fit method (BFM). The results obtained from VMI are compared with the calculated K value by Shalaby. A good agreement between two is obtained which supports our proposed model. All data for SD bands is taken from National Nuclear Data Center (NNDC) site.

Keywords: Super deformation (SD), Variable Moment of Inertia (VMI), Angular Momentum (K Value), Softness Parameter (σ), Energy ratio (R), Best Fit Method (BFM)

I. INTRODUCTION

Variable moment of Inertia (VMI) model was first proposed by Mariscotti *et.al.,* to predict different level energies of ground state bands in even-even nuclei [1]. Since then VMI model was used to study various micro and macro level band head energies and is considered as a mathematical method for microscopic derivation of the IBM (Interacting Boson Model). VMI model can explain the general dependence of the vibrational energy, projection of angular momentum (K) softness parameter (σ) and the moment of inertia with energy E_{Ik}. Goel.A *et.al*., 1990 extended VMI model to describe rotational bands in odd-odd rare earth nuclei; with small Coriolis coupling effect [2,3].
The remarkable peculiarity of this model is its ability to generalize the rotation of higher mass nucleus. It is expressed in terms of hyper geometric functions which directly give the rotational energies or their expansion in terms of the quantity K (K +1), where K is the projection of angular momentum. This is a simple two parameter model in which each nucleus is characterized by moment of inertia and σ (softness parameter) [4]. In this model the energy of a level with angular momentum I is given by the sum of a potential energy term and rotational energy term $\hbar^2 I$ (I+1). Moment of inertia is known to increase with deformation parameter β. VMI model for energy ratios of the first 4^+ to 2^+ states of even-even nuclei gives a semi empirical description of intrinsic and transition quadrupole moments and level energies. It determines the energy ratio of first 4^+ to 2^+ states of all collective nuclei, by this ratio K can be determined in terms of softness parameter (σ).

In super-deformed (SD) bands, gamma ray energies are the only spectroscopic information available till today [5]. Projection of Angular momentum (K-value) along the symmetry axis or the lowest spin (I_f) is unknown in the SD bands. Because of the non-observation of the discrete linking transitions between the super deformed states and the low lying states at normal deformation (ND), the experimental data for the spin of the rotational bands is poor. One of the most useful information to study the SD bands requires essentially the K value and the band head spin. VMI model for odd-odd nuclei developed by Goel et al; was modified to estimate the K value or lowest spin state in super deformed nuclei in terms of gamma energy ratio. The energy ratio is expressed in terms of gamma ray energies as, R = E_γ (I + 6 to I + 2) / E_γ (I + 4 to I + 2). The ratio depends on the K and σ value. The softness parameter (σ) was obtained through a two parameter formula by best fit method (BFM). A program was developed and executed using C++. Our VMI results are in good agreement with Shalaby's results [6]. They determined the K value by drawing the dynamic and static moment of inertia against the rotational frequency for different values of spin [7]. According to them there is a critical spin below which the normal behavior of the moment of inertia is reversed. The critical spin is to be regarded as the baseline spin or the lowest spin of the superdeformed bands. This lowest spin is K value for the SD bands [8]. VMI results were compared with Shalaby's two parameter formula results as given in Table, Table 2 and Table3 represents our calculations for odd-odd and even-even nuclei in SD region.

II. METHODOLOGY

The study considered methods for calculating angular momentum (K value) in terms of gamma energies (E_γ). The softness parameter (σ) obtained by BFM and Mall man's energy ratios (R) for SD bands of odd-odd and even-even nuclei.

Band head energy (E_{IK}) of rotational bands in a VMI model is given as following

$$E_{IK} = E_K + \frac{1}{2\mathfrak{I}}\left[I(I+1) - K(K+1)\right] + \frac{1}{2}C(\mathfrak{I}_I - \mathfrak{I}_K)^2 \qquad \text{…(1)}$$

Where C is the restoring force constant related to σ. In equilibrium conditions the band head energy is represented as,

$$\frac{\partial E_{IK}(\mathfrak{I})}{\partial \mathfrak{I}} = 0$$

$$\mathfrak{I}_I = \frac{\mathfrak{I}_K}{\left\{1 - \frac{I(I+1) - K(K+1)}{2C\mathfrak{I}_I^3}\right\}} \qquad \text{…(2)}$$

Then Eq.1 becomes,

$$E_{IK}(\mathfrak{I}) = \frac{1}{2\mathfrak{I}_I}\left[I(I+1) - K(K+1)\left\{1 + \frac{I(I+1) - K(K+1)}{4C\mathfrak{I}_I^3}\right\}\right] \qquad \text{…(3)}$$

Here the band head energy has been subtracted. Softness parameter (σ) in VMI model is

$$\sigma = \frac{1}{\mathfrak{I}_I}\left[\frac{\partial \mathfrak{I}_I}{\partial I}\right] = \frac{2I+1}{2C\mathfrak{I}_I^2(3\mathfrak{I}_I - 2\mathfrak{I}_K)}$$

For ground state I = K , the equation will become

$$\sigma = \frac{2K+1}{2C\mathfrak{I}_K^3}$$

Range of validity is defined as,

$$r_{IK} = \left\{\frac{1}{3} + \frac{2}{3}\operatorname{Cosh}\left[\frac{1}{3}\operatorname{Cosh}^{-1}\left\{27 \text{ X } \frac{1}{2}\left(\frac{\sigma\, I(I+1) - K(K+1)}{(2K+1)} + \frac{2}{27}\right)\right]\right\} \qquad \text{…(4)}$$

Relationship between $R_{I=(K+N)}$ energy ratios and other parameters are given as,

$$E_{IK} = \frac{I(I+1) - K(K+1)}{4\mathfrak{I}_K}\left[\frac{(3r_{IK} - 1)}{r^2_{IK}}\right] \qquad \text{…(5)}$$

For energy in state 2 is, I = K + 2,

$$E_{I=K+2} = \frac{2K+3}{4\mathfrak{I}_k}\left[\frac{(3r_{I=K+2} - 1)}{r^2_{I=k+2}}\right]$$

The energy in state 4 is, I = K + 4,

$$E_{I=K+4} = \frac{4K+10}{4g_k}\left[\frac{(3r_{I=K+4}-1)}{r^2_{I=k+4}}\right]$$

The energy in state 6 is, I = K + 6,

$$E_{I=K+6} = \frac{6K+21}{4g_k}\left[\frac{(3r_{I=K+6}-1)}{r^2_{I=k+6}}\right]$$

Mallman's energy ratios in a SD band can be written as,

$$R\left(\frac{K+2n}{K+2}\right) = \frac{E(K+2n)}{E\,(K+2)} \qquad \text{…(6)}$$

Where K + 2n is the energy in the nth state in the band relative to the band head energy $E_{(I\,=\,K)}$. Therefore energy ratio can be expressed in terms of gamma energies for SD bands as,

$$R = \frac{E_\gamma(I+6) - E_\gamma(I+2)}{E_\gamma\,(I+4) - E_\gamma\,(I+2)} \qquad \text{…(7)}$$

On substituting with above equations the final extended VMI equation is as follow.

$$R = \frac{(6K+21)\left\{\dfrac{\mathrm{Cosh}\left[\frac{1}{3}\,\mathrm{Cosh}^{-1}\left(\sigma \times 27 \times \frac{6K+21}{2K+1}+1\right)\right]}{\left\{\frac{1}{3}+\frac{2}{3}\,\mathrm{Cosh}\left[\frac{1}{3}\mathrm{Cosh}^{-1}\left(\sigma \times 27 \times \frac{6K+21}{2K+1}+1\right)\right]\right\}^2}\right\} - (2K+3)\left\{\dfrac{\mathrm{Cosh}\left[\frac{1}{3}\mathrm{Cosh}^{-1}\left(\sigma \times 27 \times \frac{2K+3}{2K+1}+1\right)\right]}{\left\{\frac{1}{3}+\frac{2}{3}\,\mathrm{Cosh}\left[\frac{1}{3}\mathrm{Cosh}^{-1}\left(\sigma \times 27 \times \frac{2K+3}{2K+1}+1\right)\right]\right\}^2}\right\}}{(4K+10)\left\{\dfrac{\mathrm{Cosh}\left[\frac{1}{3}\,\mathrm{Cosh}^{-1}\left(\sigma \times 27 \times \frac{4K+10}{2K+1}+1\right)\right]}{\left\{\frac{1}{3}+\frac{2}{3}\,\mathrm{Cosh}\left[\frac{1}{3}\mathrm{Cosh}^{-1}\left(\sigma \times 27 \times \frac{4K+10}{2K+1}+1\right)\right]\right\}^2}\right\} - (2K+3)\left\{\dfrac{\mathrm{Cosh}\left[\frac{1}{3}\mathrm{Cosh}^{-1}\left(\sigma \times 27 \times \frac{2K+3}{2K+1}+1\right)\right]}{\left\{\frac{1}{3}+\frac{2}{3}\,\mathrm{Cosh}\left[\frac{1}{3}\mathrm{Cosh}^{-1}\left(\sigma \times 27 \times \frac{2K+3}{2K+1}+1\right)\right]\right\}^2}\right\}}$$

VMI equation directly gives the rotational energies or their expansions in terms of their quantity K (K + 1), where K is the projection of angular momentum along symmetry axis. This gives a general dependence on vibration energy and moment of inertia on energy E_{IK}. The LHS is the energy ratio R which is calculated by experimental gamma energy ratio. The RHS depends on σ and K value. The σ is obtained from BFM.

RESULTS AND DISCUSSION

Final VMI equation as mentioned in the methods section was used to calculate the projection of angular momentum (K value) and was compared with the mallmann's energy ratio (R value) as mentioned in the experimental values. The results obtained from this equation were compared with Shalaby's results as given in Table 1 and an insight concurrence was observed between the two methods. The value of softness parameter (σ) is obtained by fitting Eγ energies and I values σ by using best fit method. Previously Shalaby's has done Marquardt non linear least square fitting to obtain (σ). Shalaby determined lowest spin (I_f) and K value by drawing dynamic and static moment of inertia against the rotational frequency for different values of spin. These results were used to assign the critical spin below the normal behavior of the band and reversed. Critical spin is to be considered as baseline spin or lowest spin of super deformed bands. Our study results were compared with Shalaby's et.al results and found good agreement between the two (Table 1). We also applied our model for odd-odd and even-even SD bands. The results of odd-odd and even-even are depicted in Table 2 and Table 3.

Table 1: Comparison of K value from Shalaby's results and VMI model equation.

SD Bands	I value (expt)	σ Value (10^{-4})	R value	K value (VMI)	K value (Shalaby's)
^{58}Ni (b1)	15	9.33	2.17	8	11
^{58}Cu	9	0.206	2.24	5	6
^{59}Cu(b1)	25/2	7.36	2.15	9.5	11.5
^{61}Zn	25/2	20.33	2.12	12.5	15.5
^{62}Zn	18	50.17	2.10	16	20
^{65}Zn	25/2	2.65	2.13	11.5	18.5
^{68}Zn	17	1.45	2.10	16	16
^{84}Zr	21	10.87	2.08	20	23
^{86}Zr	23	36.46	2.08	22	25
^{89}Tc	17.5	0.153	2.09	17.5	21.5

The table compares K values obtained through VMI equation and Shalaby's method.
Annotation: 'I' value is the experimental known band head spin obtained from NNDC site. σ value is softness parameter obtained through best fit method and K value is projection of angular momentum along symmetric axis.

Table 2: Calculations in Odd-Odd SD nuclei using VMI model.

SD Bands	Eγ	I value	Para meter A	Para meter B	Para meter C	Para meter D	σ value (10^{-4})	R Value	K Value
^{194}Tl (b1)	268	12	5.9001	0.0005	$1.68x10^{-7}$	$8.54x10^{-11}$	1.69	2.123	12
^{194}Tl (b2)	209	9	6.1737	0.0009	$5.22x10^{-7}$	$4.56x10^{-10}$	2.91	2.161	9
^{194}Tl(b3)	241	10	6.3801	0.0008	$3.98x10^{-7}$	$2.99x10^{-10}$	2.50	2.139	10
^{194}Tl (b4)	220	9	6.4952	0.0010	$6.12x10^{-7}$	$5.63x10^{-10}$	3.07	2.158	9
^{194}Tl (b5)	188	8	6.2876	0.0012	$9.12x10^{-7}$	$1.04x10^{-9}$	3.81	2.168	8
^{194}Tl (b6)	207	9	6.1365	0.0010	$6.48x10^{-7}$	$6.31x10^{-1}$	3.25	2.159	9
^{164}Lu(b1)	354	13	7.1791	0.0001	$5.57x10^{-9}$	$4.65x10^{-7}$	27.85	2.139	11
^{164}Lu(b2)	374	14	6.9351	0.0002	$2.3x10^{-4}$	$3.99x10^{-6}$	57.67	2.144	10
^{152}Tb(b1)	823	31	6.8402	0.0001	$5.84x10^{-5}$	$5.12x10^{-7}$	29.23	2.052	31
^{152}Tb(b2)	801	30	6.9309	0.0001	$5.76x10^{-5}$	$4.96x10^{-7}$	28.85	2.052	30
^{132}Pr(b2)	565	12	12.457	0.0013	$5.38x10^{-7}$	$3.36x10^{-10}$	2.08	2.135	12
^{130}La(b1)	762	16	12.7207	0.0010	$3.13x10^{-7}$	$1.34x10^{-10}$	1.57	2.191	16

Annotation: Eγ is gamma energy and I is band head spin obtained from NNDC experimental data. σ value is softness parameter, K value is the angular momentum and R value is the energy ratio.

Table 3: Calculations in Even-Even SD nuclei using VMI model.

SD Bands	Eγ	I Value	Parameter A	Parameter B	Parameter C	Parameter D	Softness parameter $\sigma x10^{-4}$	R Value	K Value	Moment of inertia
^{190}Hg(b1)	317	12	7.0012	0.0008	3.63x10^{-7}	2.48x10^{-10}	2.28	2.126	12	0.0714
^{190}Hg(b2)	481	11	12.3318	0.0056	1.016x10^{-5}	2.76x10^{-10}	9.08	2.16	9	0.0405
^{190}Hg(b3)	279	14	5.1273	0.0002	3.06x10^{-4}	7.03x10^{-6}	76.6	2.125	11	0.0958
^{192}Hg(b1)	214	8	7.1009	0.0011	6.78x10^{-7}	6.28x10^{-10}	3.09	2.16	8	0.070
^{192}Hg(b2)	241	10	6.3712	0.0007	3.05x10^{-7}	2x10^{-10}	2.19	2.12	10	0.078
^{194}Hg(b1)	212	8	7.0619	0.0013	9.56x10^{-7}	1.05x10^{-9}	3.68	2.17	8	0.070
^{194}Hg(b2)	222	9	6.4997	0.0008	3.93x10^{-7}	2.90x10^{-10}	2.46	2.169	9	0.076
^{194}Hg(b3)	201	8	6.6577	0.0010	6x10^{-7}	5.41x10^{-10}	3.004	2.166	8	0.075
^{194}Pb (b1)	125	4	8.8423	0.0052	1.22x10^{-9}	4.31x10^{-4}	0.1176	2.25	4	0.056
^{194}Pb (b2)	241	10	6.4838	0.0012	8.87x10^{-7}	9.85x10^{-10}	3.70	2.14	10	0.077
^{194}Pb (b3)	261	11	6.3741	0.0010	6.24x10^{-7}	5.69x10^{-10}	3.13	2.13	11	0.078
^{196}Pb (b1)	171	6	7.7352	0.0022	2.49x10^{-6}	4.25x10^{-9}	5.68	2.2	6	0.064
^{196}Pb (b2)	205	8	6.8804	0.0014	1.19x10^{-6}	1.48x10^{-9}	4.16	2.17	8	0.072
^{196}Pb (b3)	227	9	6.7270	0.0012	8.52x10^{-7}	9.10x10^{-10}	3.56	2.15	9	0.074
^{196}Pb (b4)	405	17	6.5130	0.0007	2.48x10^{-7}	1.19x10^{-10}	2.14	2.19	7	0.076
^{198}Pb (b1)	304	12	6.6677	0.0006	2.13x10^{-7}	1.147x10^{-10}	1.79	2.12	12	0.074
^{198}Pb (b2)	281	10	7.4689	0.0013	9.04x10^{-7}	9.44x10^{-10}	3.48	2.13	10	0.066
^{198}Pb (b3)	216	8	7.2423	0.0015	1.24x10^{-6}	1.54x10^{-9}	4.14	2.17	8	0.069
^{198}Po (b1)	176	6	8.0842	0.0038	7.14x10^{-6}	2.01x10^{-8}	9.40	2.2	6	0.0618
^{168}Hf (b1)	677	21	8.4785	0.0005	1.179x10^{-7}	4.07x10^{-11}	117.94	2.06	19	0.0589
^{168}Hf (b2)	771	24	8.5548	0.0004	7.48x10^{-8}	2.09x10^{-5}	93.51	2.06	21	0.0584
^{168}Hf (b3)	811	28	7.4594	0.0001	5.36x10^{-9}	4.31x10^{-7}	26.81	2.056	28	0.0670
^{150}Gd(b1)	815	30	6.964	0.0001	5.74x10^{-5}	4.94x10^{-7}	28.71	2.055	29	0.071
^{150}Gd(b4)	688	27-	0.3683	0.0014	2.12x10^{-9}	4.85x10^{-7}	0.760	2.069	26	1.357
^{150}Gd(b5)	713	28-	0.1976	0.0016	5.05x10^{-13}	2.42x10^{-18}	0.016	2.065	28	2.530
^{150}Gd(b6)	772	27	7.409	0.0002	2.08x10^{-4}	3.49x10^{-6}	53.98	2.056	27	0.0674
^{152}Dy(b1)	602	24	6.4144	0.0001	6.23x10^{-5}	5.82x10^{-7}	31.17	2.069	23	0.0779
^{152}Dy(b2)	826	34	6.2106	0.0001	6.43x10^{-5}	5.82x10^{-7}	31.17	2.069	23	0.0805
^{152}Dy(b3)	793	36	5.3630	0.0001	7.45x10^{-5}	8.33x10^{-10}	37.29	2.06	25	0.0932
^{154}Er(b1)	696	24	7.4458	0.0002	2.14x10^{-4}	3.46x10^{-6}	53.72	2.06	24	0.0671
^{154}Er(b2)	745	26	7.5218	0.0002	2.12x10^{-4}	3.39x10^{-6}	53.17	2.056	26	0.0664
^{132}Ce(b1)	770	16	12.182	0.0009	2.77x10^{-7}	1.16x10^{-10}	1.47	2.1	16	0.0410
^{134}Nd(b1)	668	17	9.6012	0.0002	1.66x10^{-4}	2.08x10^{-6}	41.66	2.10	14	0.0520
^{134}Nd(b2)	726	18	10.5296	0.0007	1.83x10^{-7}	2.88x10^{-10}	132.9	2.07	17	0.0474
^{136}Nd(b1)	657	17	9.9599	0.0003	3.61x10^{-4}	6.44x10^{-6}	60.24	2.10	13	0.0502

Annotation: Eγ is gamma energy and I is band head spin obtained from NNDC experimental data. σ value is softness parameter, K value is the angular momentum and R value is the energy ratio.

CONCLUSION

For super deformed nuclei gamma ray energies are the only spectroscopic information available. Our study considers a theoretical adaptation of VMI equation to predict angular momentum (K values) in different super deformed regions. The VMI model as proposed by Mariscotti et.al. permits calculation of angular momentum which is the sum of potential energy terms and rotational energy terms [1]. Mallmann's energy ratio (R value) relation was formulated in terms of σ and K value of nucleus. We have developed a complete model to give more information about lower spin or K value for SD bands so the spectroscopy can be done in this region. It will provide a platform for many theoreticians to find out a perfect model to relate all the data to conclude a model which could explain the features of super deformed bands. VMI model was used for odd-odd and even-even nuclei by many researchers. For SD bands transition energies are not known. So we modified the VMI model in terms of gamma energies. For the even-even and odd-odd nuclei, R2, R4, R6... are defined in terms of transition energies on the similar ground these energy ratios are calculated in terms of gamma energies. The gamma energies relation is formulated. The σ is given as a parameter. The value of sigma is obtained by fitting Eγ energies with spin using BFM in simple model four parameter formulas.

Attempts were made to compare angular momentum with Marquardt method as proposed by Shalaby [6]. Results are summarized in Table 1. Each nucleus was characterized by two adjustable parameters moment of inertia and softness parameter. These parameters were found to vary smoothly depending upon N and Z of an atom. This is the proposed model for determination of K value for SD bands. We are in a process of developing a complete model to give more information regarding SD bands so the spectroscopy can be done in this region. It will provide a platform for many theoreticians to find out a perfect model to relate all the data to conclude a model which could explain the features of super deformation bands.

LIMITATIONS OF STUDY

This study is a theoretical adaptation of VMI equation to predict K value in terms of softness parameter (σ) and energy ratio (R value) for super deformed bands. The results were compared with experimental values provided in NNDC database and therefore highly dependent on NNDC methodology. A software program was generated for computation and calculations. Therefore this model requires validation through experimental confirmation.

ACKNOWLEDGEMENT

We are thankful to Vishal Gupta from Wipro Technologies, to extend his help in furnishing ideas (through editing), which we wanted to present in this research.

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